INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2011-12
Statistics - IV, Midterm Examination, September 19, 2011
Marks are shown in square brackets. Total Marks: 50

1. Suppose $\mathbf{X} \sim N_p(\mu, \Sigma)$ where $\Sigma = \sigma^2(I_p + \rho\mathbf{1}\mathbf{1}')$ with $\sigma^2 > 0$ and $\rho > 0$.
(a) Show that $\sigma(I_p + \alpha\mathbf{1}\mathbf{1}')$ is a square root of Σ if $\alpha = (\sqrt{1+p\rho} - 1)/p$.
(b) Find the probability distribution of

$$\mathbf{Z} = \frac{1}{\sigma}\left(I_p - \frac{\alpha}{1+p\alpha}\mathbf{1}\mathbf{1}'\right)(\mathbf{X} - \mu).$$

(c) Show that $\mathbf{Z}'\mathbf{Z} \sim \chi^2$ and find its degrees of freedom. [15]

2. Consider an $I \times J$ contingency table where the (i, j) cell has probability p_{ij}. Find the maximum likelihood estimate of p_{ij}
(a) when no restrictions are placed on the row and column factors;
(b) when it is known that the row and column factors are independent. [10]

3. Suppose X_1 and X_2 are i.i.d. $N(\mu, \sigma^2)$, $U \sim \text{Exp}(1)$ and is independent of X_1. Let $Y = X_1 + U$. Show that Y is stochastically larger than X_2. [5]

4. The following table classifies a random sample of 117 couples according to height of husband and wife.

	Wife, Tall	Wife, Medium
Husband, Tall	18	28
Husband, Medium	20	51

(a) Provide a measure of association between the two factors.
(b) What features does this measure (in general) have in comparison with common measures of association for measurement data. [10]

5. Suppose $D_1, D_2, \ldots, D_n$ are continuous random variables which are independent and are symmetric about 0. Let I_j be the indicator variable which is defined as $I_j = 1$ if $D_j \geq 0$ and 0 otherwise, for $1 \leq j \leq n$.
Show that $(|D_1|, |D_2|, \ldots, |D_n|)$ and $(I_1, I_2, \ldots, I_n)$ are independently distributed. [10]